Examples related to Different Economic Problems:

1. Find the marginal revenue (MR) function of the firm, given the average revenue function -

$AR = 5 - 2Q$

Solution: It is given that

$$AR = 5 - 2Q$$

$\therefore$ Total Revenue function $(TR) = AR \cdot Q$

$$= (5-2Q) \cdot Q$$

$$= 5Q - 2Q^2$$

$\therefore$ Marginal Revenue $(MR) = \frac{d}{dQ}(TR)$

$$= \frac{d}{dQ}(5Q - 2Q^2)$$

$$= \frac{d}{dQ}(5Q) - \frac{d}{dQ}(2Q^2)$$

$$= 5 - 4Q$$

2. The total cost function of a firm is given by $TC = 5q^3 - 20q^2 + 5q + 100$

where q is the quantity produced.

(a) Find the marginal cost function.

(b) Find the level of q, where marginal cost equal to average variable cost.

Solution: It is given that

$$TC = 5q^3 - 20q^2 + 5q + 100$$

(a) Marginal cost $(MC) = \frac{d}{dq}(TC)$

$$= \frac{d}{dq}(5q^3 - 20q^2 + 5q + 100)$$

$$= 15q^2 - 40q + 5$$

(b) The total variable cost (TVC) function in the given total cost (TC) function is

$$TVC = 5q^3 - 20q^2 + 5q \qquad (\because TC = TVC + TFC)$$

$\therefore$ Average Variable Cost (AVC) $= \frac{TVC}{q}$

$$= \frac{5q^3 - 20q^2 + 5q}{q}$$

$$= 5q^2 - 20q + 5$$

The given condition is

$$MC = AVC$$

$$\Rightarrow 15q^2 - 40q + 5 = 5q^2 - 20q + 5$$

$$\Rightarrow 15q^2 - 5q^2 = 40q - 20q + 5 - 5$$

$$\Rightarrow 10q^2 = 20q$$

$$\Rightarrow 10q = 20$$

$$\Rightarrow q = \frac{20}{10} = 2$$

$\therefore$ When $q = 2$, the marginal cost equal to average variable cost.

3. The average revenue function of a firm is given by

$$AR = 200 + 5q$$

find the slope of the total revenue function.

Solution: It is given that

$$AR = 200 + 5q$$

$\therefore$ Total Revenue (TR) $= AR \cdot q = (200 + 5q) \cdot q$

$$= 200q + 5q^2$$

$\therefore$ Slope of TR function $= \frac{d}{dq}(TR)$

$$= \frac{d}{dq}(200q + 5q^2)$$

$$= 200 + 10q$$

4. The total revenue function of a firm is given by $TR = 200q - 5q^2$

Find out the elasticity of demand when $q = 2$. Also comment on the type of the commodity.

Solution: It is given that

$$TR = 200q - 5q^2$$

We know that,

$$\text{elasticity of demand } (|e_d|) = \frac{AR}{AR - MR} \quad \text{—— (1)}$$

$$\therefore \text{Average Revenue } (AR) = \frac{TR}{q} = \frac{200q - 5q^2}{q}$$

$$= 200 - 5q$$

$$\text{Marginal Revenue } (MR) = \frac{d}{dq}(TR)$$

$$= \frac{d}{dq}(200q - 5q^2)$$

$$= 200 - 10q$$

Now putting the values of AR and MR in equation (1), we have –

$$|e_d| = \frac{200 - 5q}{(200 - 5q) - (200 - 10q)}$$

$$= \frac{200 - 5q}{200 - 5q - 200 + 10q}$$

$$= \frac{200 - 5q}{5q}$$

when, $q = 2$, the elasticity of demand will be –

$$|e_d| = \frac{200 - 5.2}{5.2} = \frac{200 - 10}{10} = \frac{190}{10} = 19$$

Since $|e_d| > 1$, hence the commodity is likely to be a luxury good.