Thus, the increase in the rate of tax (t) will increase the tax revenue (T) in the above market situation.

(d) Income Tax Rate and Income Multiplier:

Income multiplier is a measure of change in national income (Y) caused by change in investment (I_0). If national income (Y) is a function of Investment (I_0) such that $Y = f(I_0)$, then the derivative of national income (Y) with respect to investment (I_0), i.e. $\frac{dY}{dI_0}$. is also termed as income multiplier (K).

Thus, the relationship between national income (Y) and investment (I_0) can be explained with the help of income multiplier (K). To demonstrate this relationship, let us assume a national income model as –

$$\left.\begin{aligned} Y &= C + I_0 + G_0 \\ C &= a + cY \quad , \quad 1 > c > 0 \end{aligned}\right\} \text{—— (1)}$$

where, Y is national income, C = consumption, I_0 is investment, G_0 is autonomous government expenditure and c is marginal propensity to consume.

Now substituting the consumption function (C) in the national income (Y) equation, we have –

$$Y = a + cY + I_0 + G_0$$

$$\Rightarrow Y - cY = a + I_0 + G_0$$

$$\Rightarrow Y(1-c) = a + I_0 + G_0$$

$$\Rightarrow Y = \frac{a}{1-c} + \frac{1}{1-c} I_0 + \frac{1}{1-c} G_0 \text{ —— (2)}$$

To get the effect of change in investment (I_0) on national income (Y), it is necessary to differentiate Y with respect to I_0, which gives –

$$\frac{dY}{dI_0} = 0 + \frac{1}{1-c} + 0 \quad [\text{since } a, c \text{ and } G_0 \text{ are constant}]$$

$$\Rightarrow \frac{dY}{dI_0} = \frac{1}{1-c} \text{ —— (3)}$$

$$\Rightarrow \frac{dY}{dI_0} = \frac{1}{s} \text{ —— (4)}$$

[since, Marginal Propensity to consume (c) + Marginal Propensity to Save (s) = 0 hence $s = 1 - c$]

If, consumption is considered as a function of disposable income ($Y_d = Y - T$) instead of gross income (Y), in the form -

$$C = a + c.Y_d \quad \text{——— (5)}$$

Now substituting the consumption function [Equation (5)], in the national income equation ($Y = C + I_0 + G_0$), we have

$$Y = a + cY_d + I_0 + G_0 \quad \text{——— (6)}$$

$$\Rightarrow Y = a + c.(Y - T) + I_0 + G_0 \quad \text{——— (7)}$$

Since $Y_d = Y - T$, where T is total income tax and it is proportional to income such that

$T = tY$, where t is the rate of income tax ——— (8)

Now substituting $T = tY$ in equation (7), we have -

$$Y = a + cY - cT + I_0 + G_0$$

$$\Rightarrow Y = a + cY - ctY + I_0 + G_0$$

$$\Rightarrow Y - cY + ctY = a + I_0 + G_0$$

$$\Rightarrow Y(1 - c(1-t)) = a + I_0 + G_0$$

$$\Rightarrow Y = \frac{a + I_0 + G_0}{1 - c(1-t)} \quad \text{——— (9)}$$

$$\Rightarrow Y = \frac{a}{1-c(1-t)} + \frac{1}{1-c(1-t)} I_0 + \frac{1}{1-c(1-t)} . G_0 \quad \text{——— (10)}$$

In order to derive the effect of change in investment (I_0) on national income (Y), the derivative of Y with respect to I_0, i.e. income multiplier $\frac{dY}{dI_0}$ is calculated as -

$$\frac{dY}{dI_0} = \frac{1}{1-c(1-t)} \quad \text{[since } a, G_0, c \text{ and } t \text{ are constant]}$$

$$\text{or } K = \frac{1}{1 - c + ct} \quad \text{——— (11)}$$

In order to trace the effect of change in income tax rate (t) on the income multiplier (K), it is necessary to differentiate k with respect to t such that

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{1}{1-c+ct}\right)$$

$$= \frac{d}{dt}\left[(1-c+ct)^{-1}\right]$$

$$= (-1)(1-c+ct)^{-1-1} \cdot \frac{d}{dt}(1-c+ct)$$

$$= -\frac{1}{(1-c+ct)^2}(0-0+c) \quad [\text{since } c \text{ is constant}]$$

$$= (-c) \cdot \frac{1}{(1-c+ct)^2}$$

$$= (-c) \cdot k^2 \qquad \left[\because k = \frac{1}{1-c+ct}\right]$$

$$= -ck^2 \qquad \text{——— (12)}$$

Thus, $\frac{dk}{dt} < 0$ and is equal to square of multiplier (k) itself times the marginal propensity to consume (c).

Finally, $\frac{dk}{dt} = \frac{d}{dt}\left[\frac{dY}{dI_0}\right]$, where $k = \frac{1}{1-c+ct} = \frac{dY}{dI_0}$

which indicates that the effect of change in income tax rate (t) on income multiplier (k) is the derivative with respect to the tax rate of the derivative of income with respect to investment.