$$\Rightarrow T = \frac{abt + adt - abt - bct}{b+d} - \frac{bd}{b+d} t^2$$

$$\Rightarrow T = \frac{adt - bct}{b+d} - \frac{bd}{b+d} t^2$$

$$\Rightarrow T = \frac{(ad-bc)t}{b+d} - \frac{bd}{b+d} t^2$$

$$\Rightarrow T = \frac{ad-bc}{b+d} \cdot t - \frac{bd}{b+d} \cdot t^2 \quad \text{——— (5)}$$

In order to derive the effect of the change in the rate of sales tax or excise duty (t) on tax yield (T), tax yield (T) has to be differentiate with respect to the rate of tax (t) such that -

$$\frac{dT}{dt} = \frac{d}{dt}\left[\frac{ad-bc}{b+d} t\right] - \frac{d}{dt}\left[\frac{bd}{b+d} t^2\right]$$

$$\Rightarrow \frac{dT}{dt} = \frac{ad-bc}{b+d} - \frac{bd}{b+d} \cdot 2t$$

$$\Rightarrow \frac{dT}{dt} = \frac{ad-bc}{b+d} - \frac{2bdt}{b+d} \quad (6)$$

Thus, it can be concluded from equation 6 that -

If $(ad-bc) > 2bdt$, then $\frac{dT}{dt} > 0$

and $(ad-bc) < 2bdt$, then $\frac{dT}{dt} < 0$.

Example:

(1) A market model is given as -

$D = 90 - 5P$

$S = -10 + 5(P-t)$

$D = S$

Verify whether an increase in the rate of tax (t) will increase tax revenve (T), where $1 > t > 0$.

Solution: The given market model is

$$\left.\begin{aligned} D &= 90 - 5P \\ S &= -10 + 5(P-t) \\ D &= S \end{aligned}\right\} \quad \text{——— (1)}$$

where, $1 > t > 0$

Now substituting demand and supply function in the equilibrium equation, we have -

$$90 - 5P = -10 + 5(P - t)$$

$$\Rightarrow 90 - 5P = -10 + 5P - 5t$$

$$\Rightarrow 5P + 5P = 90 + 10 + 5t$$

$$\Rightarrow 10P = 100 + 5t$$

$$\Rightarrow P = \frac{100}{10} + \frac{5}{10}t$$

$$\Rightarrow P = 10 + \frac{1}{2}t \quad \text{——— (2)}$$

The total tax yield or revenue (T) is the product of the rate of tax (t) and quantity (Q), such that -

$$T = t.Q \quad \text{——— (3)}$$

$$\Rightarrow T = t.[90 - 5P] \quad (\because Q_d = 90 - 5P)$$

$$\Rightarrow T = t\left[90 - 5\left\{10 + \frac{1}{2}t\right\}\right]$$

$$\Rightarrow T = 90t - 50t - \frac{5}{2}t^2$$

$$\Rightarrow T = 40t - \frac{5}{2}t^2 \quad \text{——— (4)}$$

In order to derive the effect of the change in the rate of tax (t) on the total tax revenue (T), tax revenue (T) has to be differentiate with respect to the rate of tax (t) such that -

$$\frac{dT}{dt} = \frac{d}{dt}\left[40t - \frac{5}{2}t^2\right]$$

$$\Rightarrow \frac{dT}{dt} = 40 - \frac{5}{2}.2.t^{2-1}$$

$$\Rightarrow \frac{dT}{dt} = 40 - 5t$$

Since the rate of tax (t) is a positive fraction, i.e, $1 > t > 0$, hence $\frac{dT}{dt} > 0$.

Thus, the increase in the rate of tax (t) will increase the tax revenue (T) in the above market situation.

(d) Income Tax Rate and Income Multiplier:

Income multiplier is a measure of change in national income (Y) caused by change in investment (I_0). If national income (Y) is a function of Investment (I_0) such that $Y = f(I_0)$, then the derivative of national income (Y) with respect to investment (I_0), i.e. $\frac{dY}{dI_0}$, is also termed as income multiplier (k).

Thus, the relationship between national income (Y) and investment (I_0) can be explained with the help of income multiplier (k). To demonstrate this relationship, let us assume a national income model as –

$$\left.\begin{aligned} Y &= C + I_0 + G_0 \\ C &= a + cY, \quad 1 > c > 0 \end{aligned}\right\} \quad \text{——— (1)}$$

where, Y is national income, C = consumption, I_0 is investment, G_0 is autonomous government expenditure and c is marginal propensity to consume.

Now substituting the consumption function (C) in the national income (Y) equation, we have –

$$Y = a + cY + I_0 + G_0$$

$$\Rightarrow Y - cY = a + I_0 + G_0$$

$$\Rightarrow Y(1-c) = a + I_0 + G_0$$

$$\Rightarrow Y = \frac{a}{1-c} + \frac{1}{1-c} I_0 + \frac{1}{1-c} G_0 \quad \text{——— (2)}$$

To get the effect of change in investment (I_0) on national income (Y), it is necessary to differentiate Y with respect to I_0, which gives –

$$\frac{dY}{dI_0} = 0 + \frac{1}{1-c} + 0 \quad \text{[since } a, c \text{ and } G_0 \text{ are constant]}$$

$$\Rightarrow \frac{dY}{dI_0} = \frac{1}{1-c} \quad \text{——— (3)}$$

$$\Rightarrow \frac{dY}{dI_0} = \frac{1}{s} \quad \text{——— (4)}$$

[since, Marginal Propensity to consume (c) + Marginal Propensity to Save (s) = 0 hence $s = 1-c$]