$\Rightarrow MC = f(q) + q \cdot f'(q)$

$\Rightarrow MC = AC + q \cdot f'(q)$ $\quad [\because AC = f(q)]$

$\Rightarrow MC - AC = q \cdot f'(q)$ ——— ②

where $f'(q)$ is the 1st order derivative of AC with respect to q and it represents the slope of AC curve.

When slope of AC is downward, then $f'(q) < 0$, and

$MC - AC < 0$, since $q > 0$

or $MC < AC$, implying that when AC is sloping downward, AC is greater than MC.

Similarly, when slope of AC is upward, then $f'(q) > 0$, and

$MC - AC > 0$, since $q > 0$

or $MC > AC$, indicating that when AC is sloping upward, MC is greater than AC.

Finally, when slope of AC is zero, then $f'(q) = 0$ and MC cuts AC at its lowest point, and

$MC - AC = 0$, since $q > 0$

or $MC = AC$, implying that AC and MC are equal at the minimum point of AC or when slope of AC is zero.

(c) Tax yield in Competitive Market:

The concept of derivative can be used to demonstrate the effect of change in the rate of tax (t) on the yield of sales tax or excise duties (T). To analyse this effect, a market model is considered such that -

$$\left.\begin{array}{l} \text{Demand function, } D = a - bP \\ \text{Supply function, } S = -c + dP \\ \text{Equilibrium condition, } D = S \end{array}\right\} \text{——— ①}$$

where P is the price of the product and a, b, c and d are parameters.

It is assumed that the government has introduced a sale tax or excise duty at a rate 't' per unit of output. As a result of introduction of the sales tax or excise duty, the supply price will look like $(P-t)$ because the supplier gets $(P-t)$ as price for their product after remitting 't' per unit of output to the government as tax.

After introduction of sales tax or excise duty at a rate 't', the above given market model looks like -

$$\left.\begin{aligned} D &= a - bP \\ S &= -c + d(P-t) \\ D &= S \end{aligned}\right\} \text{——} (2)$$

Now, substituting value of demand (D) and supply (S) in the equilibrium function we have -

$$a - bP = -c + d(P-t)$$

$$\Rightarrow a - bP = -c + dP - dt$$

$$\Rightarrow bP + dP = a + c + dt$$

$$\Rightarrow P(b+d) = (a+c) + dt$$

$$\Rightarrow P = \frac{(a+c) + dt}{(b+d)}$$

$$\Rightarrow P = \frac{a+c}{b+d} + \frac{d}{b+d} \cdot t \quad \text{——} (3)$$

The total tax yield (T) is the product of the rate of tax (t) and quantity demanded (q), such that -

$$T = t.q \quad \text{——} (4)$$

$$\Rightarrow T = t.[a - bP] \qquad [\text{From eq}^n (2),\ q_d = a - bP]$$

$$\Rightarrow T = t\left[a - b\left\{\frac{a+c}{b+d} + \frac{d}{b+d} t\right\}\right]$$

$$\Rightarrow T = at - \frac{b(a+c)}{b+d} . t - \frac{bd}{b+d} t^2$$

$$\Rightarrow T = \frac{at(b+d) - bt(a+c)}{b+d} - \frac{bd}{b+d} t^2$$

$$\Rightarrow T = \frac{abt + adt - abt - bct}{b+d} - \frac{bd}{b+d}t^2$$

$$\Rightarrow T = \frac{adt - bct}{b+d} - \frac{bd}{b+d}t^2$$

$$\Rightarrow T = \frac{(ad-bc)t}{b+d} - \frac{bd}{b+d}t^2$$

$$\Rightarrow T = \frac{ad-bc}{b+d}.t - \frac{bd}{b+d}.t^2 \quad \text{——— (5)}$$

In order to derive the effect of the change in the rate of sales tax or excise duty (t) on tax yield (T), tax yield (T) has to be differentiate with respect to the rate of tax (t) such that -

$$\frac{dT}{dt} = \frac{d}{dt}\left[\frac{ad-bc}{b+d}t\right] - \frac{d}{dt}\left[\frac{bd}{b+d}t^2\right]$$

$$\Rightarrow \frac{dT}{dt} = \frac{ad-bc}{b+d} - \frac{bd}{b+d}.2t$$

$$\Rightarrow \frac{dT}{dt} = \frac{ad-bc}{b+d} - \frac{2bdt}{b+d} \quad \text{(6)}$$

Thus, it can be concluded from equation 6 that -

If $(ad-bc) > 2bdt$, then $\frac{dT}{dt} > 0$

and $(ad-bc) < 2bdt$, then $\frac{dT}{dt} < 0$.

Example:

(1) A market model is given as -

$D = 90 - 5P$

$S = -10 + 5(P-t)$

$D = S$

Verify whether an increase in the rate of tax (t) will increase tax revenve (T), where $1 > t > 0$.

Solution: The given market model is

$$\left.\begin{aligned} D &= 90 - 5P \\ S &= -10 + 5(P-t) \\ D &= S \end{aligned}\right\} \text{——— (1)}$$

where, $1 > t > 0$