$$\Rightarrow MR = AR\left[1 - \frac{1}{|e_d|}\right] \qquad [\because AR = P]$$

where $|e_d|$ is the absolute value of elasticity.

$$\therefore MR = AR - \frac{AR}{|e_d|}$$

$$\Rightarrow \frac{AR}{|e_d|} = AR - MR$$

$$\Rightarrow |e_d| = \frac{AR}{AR - MR} \quad \text{——— (3)}$$

This is the relationship between average revenue (AR), marginal revenue (MR) and elasticity of demand (e_d). From this relationship it can be expressed that –

(i) If the demand for the commodity is elastic $e_d > 1$, then $MR > 0$ implying that total revenue (TR) will increase (or decrease) as q increases (or decreases), i.e, TR will move in the same direction with q as q changes.

(ii) If the demand is unitary elastic, $e_d = 1$, then $MR = 0$ indicating that TR will remain constant irrespective of whether q increases or decreases.

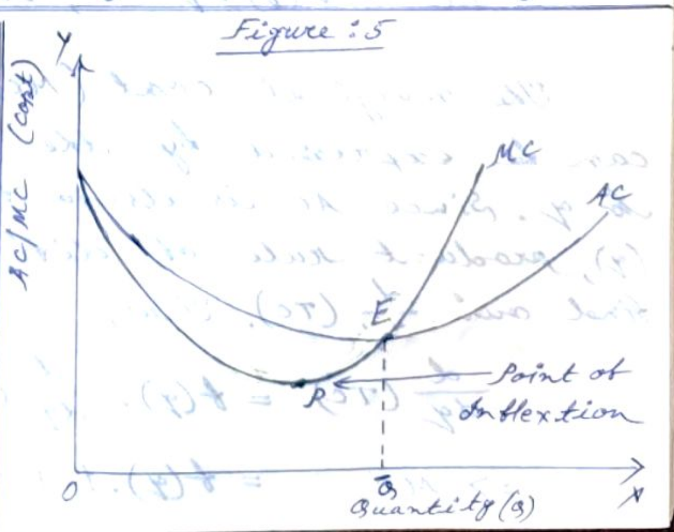
(iii) If the demand is inelastic, $e_d < 1$, then $MR < 0$ implying that TR will increase (or decrease) as q decreases (or increases), i.e. TR will move in the opposite direction with q as q changes.

(b) Relationship between Average Cost and Marginal Cost:

The relationship between Average Cost (AC) and Marginal Cost (MC) can be derived by using the product rule of differentiation.

There is a distinct relationship exists between Average cost (AC) and Marginal cost (MC), which can be displayed with the help of the Figure 5.

Figure : 5



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It is known that MC cuts AC from below and at the lowest point of AC, and it gives the optimum level of output ($\bar{q}$). From this relationship, three distinct points can be derives, such as

(a) At the left hand side of optimum level of output ($\bar{q}$), AC is sloping downward and MC is less than AC. It indicates that at this segment both AC and MC decrease upto the point of inflextion (P), but MC decreases at a faster rate as compared to AC.*

(b) At the optimum level of output ($\bar{q}$), MC cuts the AC curve at it's minimum point from below, i.e., MC = AC at this point.

(c) At the right hand side of $\bar{q}$, AC is sloping upward and MC will be greater than AC. It indicates that at this segment both AC and MC increase, but MC increases at a higher rate as compared to AC.

-* The point of inflextion is the minimum point of MC(P). Between this point (P) and minimum point of AC (E), MC starts increasing but AC is still decreasing and MC is less than AC.

The relationship between AC and MC can be derived from an economic relationship, i.e., total cost (TC) is the product of average cost (AC) and quantity produced (q), where AC is a function of quantity produced (q), i.e., $AC = f(q)$. It can be expressed as

$$TC = AC \times \text{quantity produced}$$

$$\Rightarrow TC = f(q) \cdot q \quad , \text{ where } AC = f(q) \quad \text{— (I)}$$

The marginal cost (MC) of the above total cost (TC) can be expressed by the derivative of TC with respect to q. Since AC is also a function of quantity produced (q), product rule of differentiation can be applied to find out $\frac{d}{dq}(TC)$. Thus,

$$\frac{d}{dq}(TC) = f(q) \cdot \frac{d}{dq}(q) + q \cdot \frac{d}{dq}\left[f(q)\right]$$

$$\Rightarrow MC = f(q) \cdot 1 + q \cdot f'(q)$$

$\Rightarrow MC = f(q) + q \cdot f'(q)$

$\Rightarrow MC = AC + q \cdot f'(q)$ $\quad [\because AC = f(q)]$

$\Rightarrow MC - AC = q \cdot f'(q)$ ——— ②

where $f'(q)$ is the 1st order derivative of AC with respect to q and it represents the slope at AC curve.

When slope of AC is downward, then $f'(q) < 0$, and

$MC - AC < 0$, since $q > 0$

or $MC < AC$, implying that when AC is sloping downward, AC is greater than MC.

Similarly, when slope of AC is upward, then $f'(q) > 0$, and

$MC - AC > 0$, since $q > 0$

or $MC > AC$, indicating that when AC is sloping upward, MC is greater than AC.

Finally, when slope of AC is zero, then $f'(q) = 0$ and MC cuts AC at its lowest point, and

$MC - AC = 0$, since $q > 0$

or $MC = AC$, implying that AC and MC are equal at the minimum point of AC or when slope of AC is zero.

(c) Tax yield in Competitive Market:

The concept of derivative can be used to demonstrate the effect of change in the rate of tax (t) on the yield of sales tax or excise duties (T). To analyse this effect, a market model is considered such that -

$$\left.\begin{array}{l} \text{Demand function, } D = a - bP \\ \text{Supply function, } S = -c + dP \\ \text{Equilibrium condition, } D = S \end{array}\right\} \text{——— ①}$$

where P is the price of the product and a, b, c and d are parameters.

It is assumed that the government has introduced a