(8) Application of Derivatives in Economics:

Differential calculus is generally used to deal with different economic problems. A few of them are as follows –

(a) Relationship between AR, MR and elasticity:

The relationship between Average Revenue (AR), Marginal Revenue (MR) and price elasticity of demand (e_d) can be derived by using the product rule of differentiation.

It can be derived from an economic relationship, i.e., total revenue (TR) is the product of price or average revenue (p or AR) and quantity sold (q). Symbolically,

$$TR = p.q, \text{ where } p = f(q) \quad \text{— (1)}$$

The marginal revenue (MR) of the above total revenue (TR) can be derived by the derivative of TR with respect to q. Since price (p) is also a function of quantity sold (q), product rule of differentiation can be applied to find out $\frac{d}{dq}(TR)$.

Thus, $\frac{d}{dq}(TR) = p.\frac{d}{dq}(q) + q.\frac{d}{dq}(p)$ [applying product rule]

$$\Rightarrow MR = p + q.\frac{dp}{dq}$$

$$\Rightarrow MR = p\left[1 + \frac{q}{p}.\frac{dp}{dq}\right]$$

$$\Rightarrow MR = p\left[1 + \frac{1}{\frac{dq}{dp}.\frac{p}{q}}\right]$$

$$\Rightarrow MR = p\left[1 + \frac{1}{e_d}\right] \quad \left[\because e_d = \frac{dq}{dp}.\frac{p}{q}\right] \text{— (2)}$$

There is an inverse or opposite relationship exists between price (p) and quantity (q), hence price elasticity of demand is generally negative. Hence the above relationship (equation 2) can be expressed as –

$$MR = p\left[1 - \frac{1}{|e_d|}\right]$$

$$\Rightarrow MR = AR\left[1 - \frac{1}{|e_d|}\right] \qquad [\because AR = P]$$

where $|e_d|$ is the absolute value of elasticity.

$$\therefore MR = AR - \frac{AR}{|e_d|}$$

$$\Rightarrow \frac{AR}{|e_d|} = AR - MR$$

$$\Rightarrow |e_d| = \frac{AR}{AR - MR} \quad \text{——— (3)}$$

This is the relationship between average revenue (AR), marginal revenue (MR) and elasticity of demand (e_d). From this relationship it can be expressed that –

(i) If the demand for the commodity is elastic $e_d > 1$, then $MR > 0$ implying that total revenue (TR) will increase (or decrease) as q increases (or decreases), i.e, TR will move in the same direction with q as q changes.

(ii) If the demand is unitary elastic, $e_d = 1$, then $MR = 0$ indicating that TR will remain constant irrespective of whether q increases or decreases.

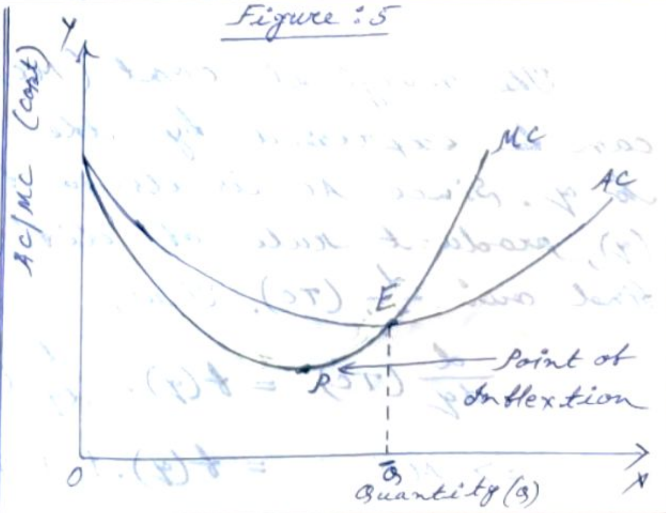
(iii) If the demand is inelastic, $e_d < 1$, then $MR < 0$ implying that TR will increase (or decrease) as q decreases (or increases), i.e. TR will move in the opposite direction with q as q changes.

(b) Relationship between Average cost and Marginal cost:

The relationship between Average Cost (AC) and Marginal Cost (MC) can be derived by using the product rule of differentiation.

There is a distinct relationship exists between Average cost (AC) and Marginal cost (MC), which can be displayed with the help of the Figure 5.

Figure : 5



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