## Matrix Operations:

### (a) Equality of Matrices:

Two matrices A and B are said to be equal if and only if

1. The order or dimension of both the matrices, i.e., A and B, are the same, and
2. Each and every element in corresponding locations of A and B has the same value.

Symbolically, A is a matrix of order $(i \times j)$ and B is a matrix of order $(i \times j)$, then $A = B$, if $a_{ij} = b_{ij}$ for all values of i and j.

For example:

If $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}_{(2\times2)}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}_{(2\times2)}$

only then $A = B$.

If $A = \begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}_{(2\times2)}$ and $B = \begin{bmatrix} 4 & 2 \\ 5 & -3 \end{bmatrix}_{(2\times2)}$

then $A \neq B$.

### (b) Addition of Matrices:

Two matrices (say A and B) can be added if and only if (1) The order of both the matrices (i.e., A and B) are the same, and

(2) It gives a new matrix (say P) of same order of the two original matrices (i.e., A and B), whose elements are the algebric sum of the corresponding elements of (A and B) the two given original matrices.

~~For~~ Symbolically, if $A = [a_{ij}]_{(i\times j)}$ $B = [b_{ij}]_{(i\times j)}$

$$\therefore (A+B) = [a_{ij}]_{(i\times j)} + [b_{ij}]_{(i\times j)}$$

$$\Rightarrow P = [p_{ij}], \text{ where } p_{ij} = a_{ij} + b_{ij}$$



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For example,

$$A = \begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}_{(2\times 2)} \text{ and } \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{(2\times 2)}$$

$$\therefore A+B = P = \begin{bmatrix} (5+1) & (2+2) \\ (1+3) & (0+4) \end{bmatrix}_{(2\times 2)} = \begin{bmatrix} 6 & 4 \\ 4 & 4 \end{bmatrix}_{(2\times 2)}$$

The matrix (or vector) addition satisfies the following properties:

(a) $A+B = B+A$

(b) $(A+B)+C = A+(B+C)$

(c) Subtraction of Matrices: Two matrices (say A and B) can be substracted if and only if

(A) The order of both the matrices (A and B) are the same, and

(B) The subtraction of B from A (or A from B) will give another matrix (say R or S) of the same order of the two given original matrices (A and B), whose elements are the algebric difference between the corresponding elements of A and B (or B and A).

Symbolically, if $A = [a_{ij}]_{(i\times j)}$ $B = [b_{ij}]_{(i\times j)}$

$$\therefore A-B = R = [a_{ij}]_{(i\times j)} - [b_{ij}]_{(i\times j)} = [R_{ij}]_{(i\times j)}$$

where, $R_{ij} = a_{ij} - b_{ij}$

and $B-A = S = [b_{ij}]_{(i\times j)} - [a_{ij}]_{(i\times j)} = [S_{ij}]_{(i\times j)}$

where $S_{ij} = b_{ij} - a_{ij}$

For example: if $A = \begin{bmatrix} 4 & 5 \\ 6 & 4 \end{bmatrix}_{(2\times 2)}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}_{(2\times 2)}$

$$\therefore A-B = C = \begin{bmatrix} 4 & 5 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} (4-2) & (5-1) \\ (6-3) & (4-4) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 0 \end{bmatrix}_{(2\times 2)}$$

$$\text{and } B-A = D = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} (2-4) & (1-5) \\ (3-6) & (4-4) \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ -3 & 0 \end{bmatrix}_{(2\times 2)}$$

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(d) Scalar Multiplication: It indicates a matrix is multiplied by a scalar (or a number), where each and every element of the matrix is multiplied by that scalar (or number). Symbolically, if $A = [a_{ij}]$ and matrix $A$ is multiplied by a scalar '$\lambda$', then the resultant matrix can be expressed as

$$\lambda . A = \lambda [a_{ij}] = [\lambda a_{ij}] = [a_{ij}]\lambda$$

For example, if $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ is multiplied by a scalar, say 2, then the resultant scalar matrix will be

$$2A = 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2\times2 & 2\times3 \\ 2\times1 & 2\times0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}$$