4.5. Types of Matrix:

Matrices can be termed differently on the basis of the structure and dimension of the matrix and the value of its elements. Thus, we have the following matrices:

A. Square Matrix:

A matrix is said to be square matrix, if its number of rows is just equal to its number of columns. In other words, a matrix of the order $(m \times n)$ is to be a square matrix, if $m = n$. For example,

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}_{(2\times 2)} \qquad B = \begin{bmatrix} 6 & 1 & 3 \\ 4 & 2 & 0 \\ -1 & 7 & 5 \end{bmatrix}_{3\times 3}$$

B. Rectangular Matrix:

A matrix whose number of rows is not equal to its number of columns is called a rectangular matrix. A $(m \times n)$ order matrix is said to be a rectangular matrix, if $m \neq n$. For example,

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 2 & -7 \end{bmatrix}_{(2\times 3)} \qquad B = \begin{bmatrix} 5 & 1 \\ 3 & 2 \\ -2 & 1 \\ 4 & 7 \end{bmatrix}_{(4\times 2)}$$

C. Identity or Unit Matrix:

The square matrix whose principal diagonal elements (say a_{11}, a_{22}, a_{33}, etc.) are equal to one (1) and all other elements or all other off-diagonal elements are zero (0), is called an identity matrix or unit matrix. It is symbolised by I. An identity matrix of order 2, order 3 and order n

can be denoted by,

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \text{ and } I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{n\times n}$$

It plays a similar role to the role played by number 1 in ordinary algebra. ~~For~~ In ordinary algebra, for any number x, $1 \times x = x \times 1 = x$. Similarly, for any matrix P, $IP = PI = P$, where I is an identity matrix.

Proof: Let us assume that,

$$P = \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 3 & 2 \end{bmatrix}_{3\times 2}$$

Then, $$IP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \cdot \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 3 & 2 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} (1\times 2 + 0\times 6 + 0\times 3) & (1\times 1 + 0\times 5 + 0\times 2) \\ (0\times 2 + 1\times 6 + 0\times 3) & (0\times 1 + 1\times 5 + 0\times 2) \\ (0\times 2 + 0\times 6 + 1\times 3) & (0\times 1 + 0\times 5 + 1\times 2) \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 3 & 2 \end{bmatrix}_{3\times 2} = P$$

Again, $$PI = \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 3 & 2 \end{bmatrix}_{3\times 2} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} (2\times 1 + 1\times 0) & (2\times 0 + 1\times 1) \\ (6\times 1 + 5\times 0) & (6\times 0 + 5\times 1) \\ (3\times 1 + 2\times 0) & (3\times 0 + 2\times 1) \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 2 & 1 \\ 6 & 5 \\ 3 & 2 \end{bmatrix}_{3\times 2} = P$$

Therefore, $IP = PI = P$.

Thus, in matrix multiplication, it is possible to insert or delete any identity matrix without affecting the matrix product. Symbolically if P and Q are two matrices, then, ~~##~~

$PQ = IPQ$ or PIQ, where I = Identity matrix.

D. Scalar Matrix:

A square matrix having a common scalar element in the principal diagonal and zeros in the off-diagonal positions is known as a scalar matrix. For example,

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times3} \qquad B = \begin{bmatrix} x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & x \end{bmatrix}_{4\times4}$$ (where x is a scalar element)

E. Diagonal Matrix:

A square matrix having scalar elements (different/not necessarily equal) in the principal diagonal and zeros in the off-diagonal positions is termed as a diagonal matrix. For example,

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}_{3\times3} \qquad B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times3} \qquad C = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

F. Null Matrix:

A matrix, not necessarily square, whose all elements are zero, is known as a null matrix and is denoted by 0. For example,

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3} \qquad O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}$$

It plays almost a similar role to the role played by zero(0) in ordinary algebra. In ordinary algebra, $a \times 0 = 0$ and $a + 0 = a$; similarly in matrix algebra, $A.O = O$ and $A + O = A$. But, if $AB = 0$ it doesnot imply that either A or B is a null matrix.

G. Triangular Matrix:

A square matrix whose all the elements in one side of the principal diagonal are zero and some or all the elements in the other side are non-zero is known as triangular matrix. It may be of two types - (a) Upper Triangular Matrix, (b) Lower Triangular Matrix.

For example, $A = \begin{bmatrix} 2 & 6 & -5 \\ 0 & 5 & 3 \\ 0 & 0 & 8 \end{bmatrix}_{3\times3}$ and $B = \begin{bmatrix} 4 & 2 & -1 & 7 \\ 0 & 3 & 2 & 5 \\ 0 & 0 & -3 & 8 \\ 0 & 0 & 0 & -2 \end{bmatrix}_{4\times4}$

are upper triangular matrix and $P = \begin{bmatrix} 7 & 0 & 0 \\ -2 & 5 & 0 \\ 6 & 2 & -7 \end{bmatrix}_{3\times3}$

and $Q = \begin{bmatrix} 7 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ -3 & 8 & 1 & 0 \\ 2 & 1 & 7 & 9 \end{bmatrix}_{4\times4}$ are lower triangular matrix.

H. Symmetric Matrix:

A square matrix whose elements are such that if the corresponding rows and columns are interchanged, then the matrix remains the same is known as a symmetric matrix. In other words, symmetric matrix, is a square matrix, remains the same after transposition. Symbolically, if P is a square matrix and $P = P'$ (Transpose of P), then P is a symmetric matrix. For example,

$A = \begin{bmatrix} 6 & 5 \\ 5 & 2 \end{bmatrix}_{2\times2}$ and $B = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 1 & 4 \\ 5 & 4 & 2 \end{bmatrix}_{3\times3}$ are

symmetric matrix.

Note: * Diagonal and Off-Diagonal Elements:

An element a_{ij} of a matrix is said to be diagonal element, if $i = j$, it means a_{11}, a_{22}, a_{33}, etc. are diagonal elements.

An element a_{ij} of a matrix is said to be off-diagonal element, if $i \neq j$, the elements a_{12}, a_{23}, a_{31}, etc. are off-diagonal elements.

The line consisting of diagonal elements is termed as 'principal diagonal.' Symbolically,

if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$, then the principal diagonal consists of a_{11}, a_{22} and a_{33} elements.

Matrix Operations:

(a) Equality of Matrices:

Two matrices A and B are said to be equal if and only if

1. The order or dimension of both the matrices, i.e., A and B, are the same, and
2. Each and every element in corresponding locations of A and B has the same value.

Symbolically, A is a matrix of order $(i \times j)$ and B is a matrix of order $(i \times j)$, then $A = B$, if $a_{ij} = b_{ij}$ for all values of i and j.

For example:

If $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}_{(2\times2)}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}_{(2\times2)}$

only then $A = B$.

If $A = \begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}_{(2\times2)}$ and $B = \begin{bmatrix} 4 & 2 \\ 5 & -3 \end{bmatrix}_{(2\times2)}$

then $A \neq B$.

(b) Addition of Matrices:

Two matrices (say A and B) can be added if and only if (1) The order of both the matrices (i.e., A and B) are the same, and

(2) It gives a new matrix (say P) of same order of the two original matrices (i.e., A and B), whose elements are the algebric sum of the corresponding elements of (A and B) the two given original matrices.

Symbolically, if $A = [a_{ij}]_{(i\times j)}$ $B = [b_{ij}]_{(i\times j)}$

$\therefore (A+B) = [a_{ij}]_{(i\times j)} + [b_{ij}]_{(i\times j)}$

$\Rightarrow P = [p_{ij}]$, where $p_{ij} = a_{ij} + b_{ij}$