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Matrix and Determinant

4.1. Introduction and Definition:

A matrix is a rectangular array of numbers, variables or parameters arranged in rows and columns. A matrix having 'm' rows and 'n' columns can be expressed in general form as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

(1st row, 2nd row, mth row; 1st column, 2nd column, nth column)

where, $a_{11}, a_{12}, \ldots$ etc. are elements of the matrix. Each member of the array in a matrix is termed as "element of a matrix". The elements of a matrix A can also be expressed as:

$$A = [a_{ij}], \text{ where } i = 1, 2, \ldots, m \text{ \& } j = 1, 2, \ldots, n$$

Generally, a matrix is denoted by either '[ ]' (square brackets) or '( )' (large parentheses) or '‖ ‖' (double bars).

The concept of matrix was first developed by a British Mathematician Arthur Kelly and used to solve different types of linear problems. It is a very useful mathematical instrument to deal with the problem of solving a system of simultaneous linear equations. When there are two / three equations with a small number of variables (say two or three), then the system of equations can be solved by ordinary mathematical deduction. But the ordinary mathematical deduction will fail to solve a system of equations having a large number of equations with a large number of variables and matrix algebra helps in this respect.

For example,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & -6 \end{bmatrix}_{2\times 2}, \quad B = \begin{bmatrix} 1 & 6 & 4 \end{bmatrix}_{1\times 3},$$

$$C = \begin{bmatrix} 5 & 2 \\ 7 & -8 \\ 2 & 0 \end{bmatrix}_{3\times 2}, \quad \text{etc.}$$

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4.2. Matrix Formulation:

Let us assume a linear n-equation system with 'm' variables as:

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m &= c_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m &= c_2 \\ \cdots \quad \cdots \quad \cdots \quad & \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nm}x_m &= c_n \end{aligned}\right\} \text{— (1)}$$

where, the first subscript of the coefficient 'a' attached to each variable $(x_1, x_2, \cdots x_m)$ refers to the equation number and second subscript refers to the variable. and c_i refers to the constant term of the ith equation.

a_{ij} refers to the coefficient of x_j variable in the ith equation.

The above system of equations can be expressed in a ~~small~~ compact form by using matrix algebra. It has three component values- (a) the set of coefficients (a_{ij}), (b) the set of variables $(x_1, x_2, \cdots x_m)$, and (c) the set of constants $(c_1, c_2, \cdots, c_n)$. These three sets can be expressed in rectangular arrays symbolising as A, X and C respectively. Thus,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & & a_{nm} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_m \end{bmatrix}, \quad \& \quad c = \begin{bmatrix} c_1 \\ c_2 \\ \cdots \\ c_n \end{bmatrix} \text{— (2)}$$

All these three rectangular arrays is a matrix.

A simple market model can also be expressed in matrix notation as:

$$\begin{aligned} Q_d &= a - bP \\ Q_s &= -c + dP \\ Q_d &= Q_s \end{aligned} \quad \text{— (3)}$$

where Q_d is the demand for the commodity, Q_s is the supply of the commodity, P is the price and a, c, b, and d are the parameters.

The above market model can also be ~~rewritt~~ rewritten as -

$1.Q_d + 0.Q_s + b.P = a$

$0.Q_d + 1.Q_s - d.P = -c$ ——— (4)

$1.Q_d - 1.Q_s + 0.P = 0$

The above system of equations can also be expressed in rectangular arrays, which have three sets symbolising as A, X and C respectively. Thus,

$$A = \begin{bmatrix} 1 & 0 & b \\ 0 & 1 & -d \\ 1 & -1 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} Q_d \\ Q_s \\ P \end{bmatrix} \text{ and } C = \begin{bmatrix} a \\ -c \\ 0 \end{bmatrix}$$

Each of these three rectangular arrys is a matrix.

4.3. Order or Dimension of a Matrix:

The total number of rows and total number of columns of a matrix determine the 'order or dimension' of the matrix. If a matrix has 'm' number of rows and 'n' number of columns, then the order or dimension of the matrix is $(m \times n)$. ~~For example,~~ It should be noted that the first value within the bracket should be the number of rows and second value should be the number of columns. If a matrix A, has two rows and 3 columns, then the order or dimension of A will be (2×3), which is expressed as -

$$A = \begin{bmatrix} 2 & 1 & 6 \\ 4 & -3 & 0 \end{bmatrix}_{(2\times3)} \longrightarrow \text{(Order or Dimension)}$$

4.4. Vector and Scalar:

A matrix having only one row or one column is termed as 'vector'. A matrix which has only one row, is known as row vector and which has only one column, is called column vector. For example,

$$A = \begin{bmatrix} 2 & 1 & 6 \end{bmatrix}_{(1\times3)} \qquad B = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}_{(3\times1)}$$

Row Vector — Column Vector

A matrix which has only one row and ~~only~~ one column, i.e., having only one element, is known as 'scalar'. It behaves almost like a number in ordinary algebra. For example,

$$A = [2]_{(1\times 1)}$$

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