AUTO CORRELATION / SERIAL CORRELATION :

Autocorrelation is defined as correlation between members of series of observations ordered in time or space. In other words, it refers not to the the relationship between two or more variables but rad to the relationship between successive values of the variable.

According to the assumption of OLS model,

$$\begin{aligned} \text{cov}(U_i U_j) &= E\{U_i - E(U_i)\}\{U_j - E(U_j)\} \\ &= E(U_i U_j) \\ &= E(U_i) \cdot E(U_j) \\ &= 0 \quad \text{for } i \neq j \qquad \because E(U_i) = E(U_j) = 0 \end{aligned}$$

This assumption implies that the successive value of the disturbance term U are temporarily independent. If the assumption does not satisfy i.e. if the value of U in any particular period is correlated with its own preceding values (s), then we call it an autocorrelation of the random variable.

Autocorrelation is a common phenomenon in most economic variable eg. In a study of relationship between output and inputs of a firm from monthly observations, non-autocorrelation disturbance implies that unusual happening affecting production of the firm in particular time period is strictly temporary.

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CAUSES OF AUTOCORRELATION :

The main causes which give riso to auto correlation are -

1. Omitted explanatory Variable :

This is one of the major cause of auto correlation. The

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error term represents the influence of omitted variables and because of this; an error term in one period may have a relation with the error terms in successive periods.

2. Mis specification of the Mathematical form :

Mis-specification of the relationship is another cause of auto correlation. For example, let us suppose that the investigator suspects linear relation between the dependent variable (Y) and the independent variable (X), while the true relation between them was quadratic i.e the relation containing X and X^2 terms. In such a situation, the disturbance term will contain X^2 term. Thus even if the disturbance term is non-autocorrelated but the term will be autocorrelated in this specified relation.

3. Mis specification of the True Random term 'U' :

Disturbance term may be autocorrelated because it contains errors of measurement. If the explanatory variable is re measured wrongly, the serial disturbance will be auto correlated.

CONSEQUENCES OF AUTOCORRELATION :

The various consequences of autocorrelation are ——

1. Pure serial correlation doesn't cause bias in the estimates. If the error term is serially correlated, one of the assumptions of the Gauss-Markov theorem is violated, but this violation doesn't cause the coefficient estimates to be biased. If the serial correlation is impure, however, bias may be introduced by the use of an incorrect specification.

2. Serial correlation / auto correlation causes OLS estimates to no longer be the minimum variance estimates. Although the violation of classical assumption causes no bias, it does affect the other main conclusion of the Gauss-Markov theorem, i.e. minimum variance. In particular, we can't prove that the distribution of the OLS estimators is minimum variance in the presence of auto correlation.

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3. Auto correlation or serial correlation causes the OLS estimates of the standard errors, $SE(\hat{\beta})$s to be biased, leading to unreliable hypothesis testing. With serial correlation, the OLS formula for the standard errors produces biased estimates of the $SE(\hat{\beta})$. Because the $SE(\hat{\beta})$ is a primary component in the t-statistic, these biased $SE(\hat{\beta})$s cause biased t-scores and unreliable hypothesis testing in general.

DETECTION OF AUTOCORRELATION :

1. The Von Neumann Ratio Test :

The von Von Neumann Ratio is one of the theoritical test for providing the randomness of the disturbance term. It is the mean square successive difference to the variance.

Mathematically,

$$\frac{\sigma^2}{S^2} = \frac{\sum_{i=2}^{n} (e_i - e_{i-1})^2/(n-1)}{\sum_{i=1}^{n} (e_i - \bar{e})^2/n}$$

In OLS $\bar{e}=0$. For large observations, $\frac{\sigma^2}{S^2}$ may be taken as normally distributed with

$$\text{mean} = E\left(\frac{\sigma^2}{S^2}\right) = \frac{2n}{(n-1)} \text{ and}$$

$$\text{variance} = \text{var}\left(\frac{\sigma^2}{S^2}\right) = \frac{4n^2(n}{}$$

The mean and variance are true only if the values of 'e' are independently distributed and is not true of least square residuals, even when the true disturbance, are independently distributed.

2. Durbin Watson Test :

The test was developed by J. Durbin and G. Watson in 1951 and is also called Durbin Watson t statistics and states as —

$$d = \frac{\sum_{i=2}^{n} (\hat{U}_i - \hat{U}_i - 1)^2}{\sum_{i=1}^{n} \hat{U}_i^2}$$

Durbin Watson test give us a technique which can detect the presence of auto-correlation.

To test the hypothesis of non-auto correlation, the null

hypothesis is $H_0 \rightarrow P = 0$ against the alternative hypothesis of positive autocorrelation of first order $H_1 \rightarrow P > 0$.

The d-statistics used

$$d = \frac{\sum_{i=2}^{n} (e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$$

Where 'e' is the OLS residual. Now calculated d is compared with lower and upper critical values d_L and d_U for all the values of X. The decisions are ——

(i) Reject the null hypothesis if $d < d_L$

(ii) Accept the hypothesis if $d > d_U$

(iii) The test is inconclusive if $d_L < d < d_U$

In any case the value of d exceeds 2, test the hypothesis against the alternative hypothesis of negative autocorrelation of first order. The decision will be

(i) Reject H_0 if $d > 4 - d_L$

(ii) Accept H_0 if $d < 4 - d_U$

(iii) Test is inconclusive if $4 - d_U < d < 4 - d_L$

REMEDIES OF AUTOCORRELATION :

When auto-correlated disturbance term are present in a model, we should investigate the absence of any other key explanatory variables first. If we cannot identify any such explanatory variable to eliminate autocorrelation from our model, then we need to transform the variables.