(f) Division of Matrices: In ordinary algebra, if $ax = c$, then the value of x is calculated by dividing c by a, i.e., $x = \frac{c}{a}$. But, in matrix algebra, if ~~AxC~~ $AX = C$, the value of the matrix 'X' cannot be calculated by dividing C by A, i.e., $X = \frac{C}{A}$. Instead of this solution, the value of 'X' is ~~me~~ estimated by $X = A^{-1}.C$, where A^{-1} is known as 'the inverse of matrix A'.

To calculate 'inverse of matrix A' (A^{-1}), the following formula is generally used –

$$A^{-1} = \frac{\text{Adjoint of }(A)}{\text{Determinant value of }(A)} \text{ or } \frac{\text{Adj of }(A)}{|A|}$$

where, Adjoint of (A) = Transpose of the Co-factor Matrix of A

$$= [\text{Co-factor Matrix of } A]'$$

So it is necessary to know certain concepts like determinant, transpose of matrix, identity matrix, co-factor matrix, etc. to discuss the conception of matrix inversion.

Inverse of a Matrix: The concept of inverse of a matrix is very useful in economics and generally used in solving simultaneous equation system, in input-output analysis and regression ~~stage~~ analysis. Hence the procedure of calculation of 'inverse of a matrix' should be understood well. But, the determination of inverse of a matrix is not straight forward and is a lengthy procedure. The operation of dividing one matrix directly by another does not exist in matrix theory.

In ordinary algebra, if $a \times b = 1$, then $a = \frac{1}{b}$, i.e., b is inverse or reciprocal of a or a is inverse or reciprocal of b. Thus, the product of any number (say a) and its inverse or its reciprocal (a^{-1} or $\frac{1}{a}$) ~~gives~~ is one (if $a \neq 0$). Symbolically,

$$a . \frac{1}{a} = 1 \text{ or } a . a^{-1} = a^{-1} . a = 1$$

Date
Page (20)

~~In~~ In matrix algebra, the inverse of a matrix A, denoted by A^{-1} can be measured if

(a) A is a square matrix, and

(b) $A.A^{-1} = A^{-1}A = \text{Identity Matrix } (I)$

In matrix algebra, if there is a relation like

$AX = C$

then, (or) $X = A^{-1}C$ [since $A^{-1}.A = I$]

and A^{-1} can be measured by using the following formula -

$$A^{-1} = \frac{\text{Adjoint of } (A)}{\text{Determinant value of } (A)} \text{ or } \frac{Adj\,(A)}{|A|}$$

where Adjoint of A = Transpose of Co-factor Matrix of (A)

$= [\text{Co-factor Matrix of } A]'$

As it is already noted that 'inverse of a matrix' will be exists only for the square matrices. However, it does not indicate that every square matrix has an inverse. The existence of inverse of a matrix (say A) requires that the determinant value of the matrix (symbolically by $|A|$ (say)) should be non-zero.

When a square matrix does not have an inverse, the matrix is termed to be 'singular' and when it has an inverse, the matrix is said to be "non-singular".

There are certain important properties of matrix inversion. They are as follows -

1. $(A^{-1})^{-1} = A$

2. $(AB)^{-1} = B^{-1}.A^{-1} / (ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

3. $(A')^{-1} = (A^{-1})'$